01.07.2019

Bar-Ilan University

Methods of Mathematical Physics
86-818-01

Lecturer: Dr. Jonathan Ruhman

Course type: Lecture + Practice

Date: 2019-2020 **semester:** B **weekly hours:** 2L + 2P

A. Aim of course:

Improving knowledge in differential equations and ordinary differential equations, asymptotic analysis, approximation techniques and interference theory.

Outcomes:

Upon completion of this course the student will know how to implement asymptotic methods and perturbation theory to local and global analysis of differential equations (including eigenvalue problems).

Summary:

Classification of singular points, local analysis close to an irregular singular point, asymptotic series, Stokes phenomenon. Global analysis, singular and non-singular perturbation techniques, boundary layer theory, WKB. Summation of series, Pade summation, theory of Stieltjes functions. Asymptotic expansion of integrals: Laplace method, stationary phase and the method of steepest decent.

B. Details of subjects to be covered:

Week by week:

1. General introduction, perturbative expansion, singular and non-singular perturbation.
2. Local analysis of homogenous and inhomogeneous linear equations, classification of singular and non-singular points, Fuchs and Frobenius theories, analysis close to irregular singular point (ISP).
3. Local analysis close to ISP (cont.), ISP at infinity, Airy functions, Stokes phenomenon.
4. Global analysis using WKB series, geometrical and physical optics, Regular Sturm-Liouville problems.

5. Matched asymptotic approximation, WKB with one and two turning points, eigenvalues problem and Bohr-Sommerfeld quantization.
6. Gamma function and its analytic continuation to the negative x-axis, incomplete Gamma function and Exponential integrals. Asymptotic matching of integrals and differential equations.
7. Boundary-Layer theory, inner and outer solutions, distinguished limits, examples.
8. Summation of converging series, Shanks transformation and Richardson extrapolation, Riemann zeta function.
9. Summation of divergent series, Euler and Borel summations, Pade summation.
10. Pade sequences and Stieltjes functions, moment problem, Charleman's condition.
11. Asymptotic expansion of integrals, integration by parts, Laplace method and Watson's lemma.
12. Method of stationary phase and steepest decent.

C. Course mandatories:

Bachelor's degree or at least the mathematics courses.

D. Grading:

Final exam: 80%; Homework: 20%

Bibliography:

Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory, by Carl M. Bender, Steven A. Orszag